Thank You, Emily Rencsok, for these notes.

## Electric fields

$F_e = \frac{kq_1q_2}{r^2}$

$E = \frac{F_e}{q}$ Small, positive test charge

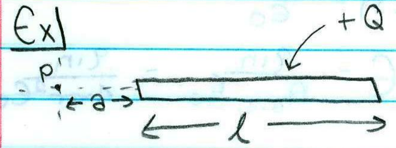
$E = \frac{kq}{r^2}$

$E = k\int \frac{dq}{r^2}\hat{r}$ continuous charge distribution

- Efield lines
  - $\oplus \rightarrow \ominus$
  - $\perp$ to surface
  - Never loops

- Charge densities
  - $\lambda = \frac{Q}{L}$
  - $\sigma = \frac{Q}{A}$
  - $\rho = \frac{Q}{V}$

Ex)




$\lambda = \frac{Q}{L} = \frac{dq}{dx}$

$dq = \lambda dx$

$E = \int \frac{kdq}{r^2} = k\int \frac{\lambda dx}{x^2}$

$E = k\lambda \int_a^{a+\ell} x^{-2}dx$

$E = k\lambda \left[\frac{-1}{x}\right]_a^{a+\ell}$

$E = -\left[\frac{k\lambda}{x}\right]_a^{a+\ell} = -\left[\frac{k\lambda}{a+\ell} - \frac{k\lambda}{a}\right]$

$E = k\lambda\left[\frac{1}{a} - \frac{1}{a+\ell}\right]$

$E = k\lambda\left(\frac{a+\ell-a}{a(a+\ell)}\right)$

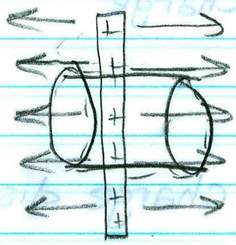
$$\boxed{E = -\frac{kQ}{a(a+\ell)}\hat{i}}$$

## E flux

$\Phi_E = \int E \cdot dA = EA\cos\theta$

$\Phi_E = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$

**Ex**



$\Phi_E = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$

$E \oint_{ends} dA \cos\theta = \frac{q_{in}}{\epsilon_0}$

$E A_{end} = \frac{q_{in}}{\epsilon_0}$

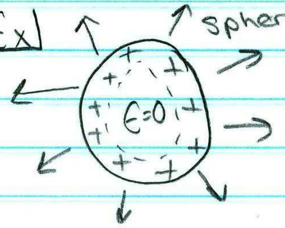
$E\, 2 A_{end} = \frac{\sigma A_{end}}{\epsilon_0}$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

$\sigma = \frac{Q}{A}$

$q_{in} = \sigma A_{end}$

**Ex** sphere




$\Phi_E = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$

$E = \frac{\sigma}{\epsilon_0}$

$E = 0$ inside

$\Phi = \oint E\, dA \cos 0 = \frac{q_{in}}{\epsilon_0}$

$r > R$

$EA = \frac{q_{in}}{\epsilon_0}$

$E = \frac{q_{in}}{A_{G.S.}\epsilon_0} = \frac{q_{in}}{4\pi r^2 \epsilon_0} = \boxed{\frac{kq_{in}}{r^2}}$

$U_{ele} = \frac{kq_1 q_2}{r}$

$\Delta V = \frac{\Delta U_{ele}}{q}$ $\quad \Delta V = \frac{kq}{r}$ $\quad \Delta V = k\int \frac{dq}{r}$ $\quad \Delta V = -Ed$ (constant E field)

$\Delta V = -\int E \cdot ds \quad \left(E = \frac{-dV}{dr}\right)$

$1\,eV = 1.60 \times 10^{-19}\,J$

$C \equiv \frac{Q}{\Delta V}$

$C_{\parallel plate} = \frac{k\epsilon_0 A}{d}$

$C_s = \left(\frac{1}{C_1} + \frac{1}{C_2} + \ldots\right)^{-1}$ $\Delta V_t = \Delta V_1 + \Delta V_2$ $Q_1 = Q_2$

$C_{\parallel} = C_1 + C_2 + \ldots$ $\Delta V_1 = \Delta V_2$ $Q_t = Q_1 + Q_2$

$U_C = \frac{Q^2}{2C} = \frac{1}{2} Q\Delta V = \frac{1}{2} C \Delta V^2$

$I = \frac{dq}{dt} = nAv_d q$

$\Delta V = IR$

$R = \frac{\rho \ell}{A}$

$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$

emf vs. $\Delta V_t$

$\Delta V_t = \mathcal{E} - Ir$

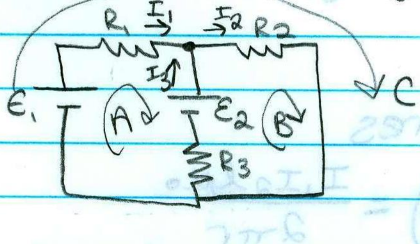
$R_{\parallel} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots\right)^{-1}$ $\Delta V_1 = \Delta V_2$ $I_t = I_1 + I_2$

$R_S = R_1 + R_2 + \ldots$ $\Delta V_t = \Delta V_1 + \Delta V_2$ $I_1 = I_2$

- Kirchhoff's Rules

1) $\Delta V_{loop} = 0$

2) $\Sigma I_{in} = \Sigma I_{out}$ of a junction




$\Delta V_A = 0 = \mathcal{E}_1 - \Delta V_{R_1} - \mathcal{E}_2 + \Delta V_{R3}$

$0 = \mathcal{E}_1 - I_1 R_1 - \mathcal{E}_2 + I_3 R_3$

$\Delta V_B = \mathcal{E}_2 - \Delta V_{R2} - \Delta V_{R3} = 0$

$0 = \mathcal{E}_2 - I_2 R_2 - I_3 R_3$

$I_1 + I_3 = I_2$

- RC Circuit $\tau = RC$ *Time for a 63.2% change
  - Derive $q$ + $I$
  - Limits
  - Charging: $q(t) = \mathcal{E}C(1 - e^{-t/RC})$ $t = 0 \rightarrow I_i = I_{max}$, $Q_i = 0$; $t \approx \infty \rightarrow I_f = 0$, $Q_f = Q_{max}$

- Ammeter
  - in series
  - small resistance
- Voltmeter
  - in parallel
  - large resistance

Magnetism

$F_B = qVB\sin\theta = ILB\sin\theta$ $F_B = IL' \times B$ $F_B$ on loop in constant B is zero

$\tau = NIA \times B = NIAB\sin\theta$

$V \perp$ to B field

$\Sigma F_{in} = F_B = ma_c$ $\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f$

- Mass spectrometer
  - V selector, $\mathcal{E}$ + B $\perp$ to each other
  - Detector $\Rightarrow$ just B

- Biot-Savart * B at center of arc/circle * Along axis of a loop

$$dB = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2}$$

- 2 current-carrying wires

$$F_B = I_1 \ell B_2 = I_1 \ell \left(\frac{\mu_0 I_2}{2\pi r}\right) = \frac{I_1 I_2 \ell \mu_0}{2\pi r}$$

same dir = attract
opp dir = repel

- Ampere's Law
  $\oint B \cdot ds = \mu_0 I_{in}$
  $B = \frac{\mu_0 I}{2\pi r}$
  $B_s = \frac{\mu_0 N I}{l}$

  * B field around current-carrying wire
  * In a solenoid
  * Inside wire

- $\oint B \cdot dA = 0 = \Phi_B$ B lines are "always" in loops

- $\mathcal{E} = -N \frac{d\Phi_B}{dt} = -N \frac{d}{dt}(BA\cos\theta)$

- Motional emf $\Delta V = vBl\sin\theta$ ← $\theta$ btwn V + B

- $\mathcal{E}_{generator} = \omega NBA\sin(\omega t)$

  $\mathcal{E}_L = -L\frac{dI}{dt} = -N\frac{d\Phi_B}{dt} \Rightarrow L = \frac{N\Phi_B}{I}$

- RL circuit $\tau = L/R$
  $t=0$ – $I_i = 0$, $\frac{dI}{dt} = max$ $\quad I_f = max$, $\frac{dI}{dt} = 0$

  $U_L = \frac{1}{2}LI^2$

- LC circuit
  - SHM
  - derive $\frac{d^2Q}{dt^2}$ $\quad \omega = \frac{1}{\sqrt{LC}}$

  $\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f$

  $Q(t) = Q_{max}$